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Albert-Ludwigs-Universität Freiburg
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Model Theory

Sheet 4

Deadline: 13.11.25, 2:30 pm.

Exercise 1 (11 points)**.**

In the language $\mathcal{L}_1 = \{E\}$ with a binary relation symbol E let T_1 be the theory stating that E is an equivalence relation with infinitely many classes such that all classes are infinite.

a) Show that T_1 is consistent, complete and has quantifier elimination.

Let $\mathcal{A}$ be a countable model of T_1 and a an arbitrary element from A.

b) Describe (informally) all 1-types in $\mathcal{A}$ over $\{a\}$.

c) Describe (informally) all 1-types in $\mathcal{A}$ over A. What is the size of $S_1^{\mathcal{A}}(A)$? Which types in $S_1^{\mathcal{A}}(A)$ are realized?

Now let $\mathcal{L}_2 = \{R\}$ be another language with a binary relation symbol and T_2 the $\mathcal{L}$-theory of the random graph, i.e. R is interpreted as the edge relation (as discussed in the lecture). Recall that T_2 is complete with quantifier elimination. Given a countable model $\mathcal{B}$ of T_2 let b be an arbitrary element of B.

d) Describe (informally) all 1-types in $\mathcal{B}$ over $\{b\}$.

e) Describe (informally) all 1-types in $\mathcal{B}$ over B. What is the size of $S_1^{\mathcal{B}}(B)$?

Exercise 2 (5 points)**.**

Let $n \geq 1$ be a natural number and $\mathcal{M}$ a model of the $\mathcal{L}$-theory T.

a) Given an n-tuple $\bar{m}$, show that $\mathrm{tp}(\bar{m}) \coloneqq \{\varphi[\bar{x}] \mid \varphi[\bar{x}]\ \mathcal{L}\text{-formula s.t. } \mathcal{M} \models \varphi[\bar{m}]\}$ is an element of $S_n(T)$, i.e. a n-type in T. Such a type is *realized in* $\mathcal{M}$.

We now assume that T is complete.

b) Show that the types realized in $\mathcal{M}$ are dense in $S_n(T)$.

Exercise 3 (4 points)**.**

Let $C \subset B$ be subsets of the universe of an $\mathcal{L}$-structure $\mathcal{A}$. As in Sheet 3, Exercise 4, consider the map $\mathrm{restr} : S_n^{\mathcal{A}}(B) \to S_n^{\mathcal{A}}(C)$.

a) Show that the map restr is continuous and closed.

b) Using Exercise 1 (of this sheet), determine whether restr is injective.

The exercise sheets can be handed in in pairs. Submit them in the mailbox 3.19 in the basement of the Mathematical Institute.